Enhancing an Ability Mathematical Reasoning through Metacognitive Strategies

by W Lestari

Enhancing an Ability Mathematical Reasoning through Metacognitive Strategies

View the article online for updates and enhancements.

Enhancing an Ability Mathematical Reasoning through Metacognitive Strategies

W Lestari[1] and Jailani[2]

[1] Graduate School of Mathematics Education Program, Yogyakarta State University
[2] Department of Mathematics Education, Yogyakarta State University

wahyu_lestari2016@student.uny.ac.id

Abstract. This study compared effects of the colaborative learning with or without metacognitive strategies on higher and lower student's achiever of the mathematical reasoning. A total of one hundred and twenty-two of grade eight (M = 13.8; SD = 0,5 year) in four heterogeneous classrooms selected from an Indonesian junior high school participated in this study. Performance measures on reasoning abilities composed by three parts: *making conjecture, providing arguments*, and *observing patterns*. The results indicated that students who were exposed to the metacognitive strategies within colaborative learning (COLAB+META) significantly outperformed their counterparts who were exposed to colaborative learning with no metacognitive strategies (COLAB). This work provided the evidences that the advantages of using metacognitive strategies to empower reasoning in mathematics. Furthermore, the findings showed the positive effects of COLAB+META method on haigher and lower achievers.

1. Introduction

The innovation for teaching mathematics redirected the teachers to focus on mathematical reasoning [1]. The importance of mathematical reasoning in mathematics education was reflected by the fact that this aspect was included in mathematics activities in the mathematics curricula in many countries (see [2]–[5]). For students, involving in mathematical reasoning, would give powerful foundation for student's understanding of many mathematical ideas, such as trigonometry [6]; rate of change and linearity [7]; exponential growth [8]; proportion [9]; variable [10]. On the other words, the reasoning skills and the mathematics materials were connected each other. Moreover, [11] stated that mathematics materials would be easily to be understood through reasoning skill, and reasoning skills could be trained through mathematics learning. Therefore, it was important for the teachers in teaching and learning mathematics paying attention with the students' reasoning skills.

In the popularity of mathematical reasoning as a student learning outcomes, there were empirical and theoretical reason for suggesting the students learn more by metacognitive processes to enhance mathematical reasoning ([1], [12]). Furthermore, this strategies should engage them to stimulate metacognition by encouraging behaviors such critical thinking, reflection, questioning, inquiry, and self-explanation [13]–[17]. This study was rooted in this approach that aimed to compared effects of metacognitive strategies approaches on the student's mathematical reasoning.

With respect to metacognitive strategies, several studies (see [18]) recomended the teachers to provide the students opportunity-to-learn to construct mathematical meaning, by involving them in

mathematical discourse through the use of self-questioning. The question like "*does my solution make sense*" was helpful for the students reflected on the reasonableness of the problem in mathematics. Other kinds of metacognitive strategies called IMPROVE (see [19]) which promoted the students to involve in the regulatory learning by focusing on: (a) *comprehensing* the problem; (b) constructing connections between previous and new knowledge; (c) using *strategies* appropriate to solve the problem; and (d) *reflecting* the processes and the solution.

Learning by matacognitive strategies might have unique benefit for the mathematics class. Most researches that had successfully used the metacognitive strategies in process of learning mathematics as an attempt to improve the students' performance on mathematical reasoning (see [12], [20]). Accordingly, present work attempted metacognitive strategies within collaborative learning for teaching mathematics in heterogeneous classrooms. It could be hypothesised that metacognitive strategies embedded in collaborative mathematics classrooms would exert positive effects in mathematical reasoning. So that, the findings indicated the differential effects of metacognitive strategies on novice and expert in mathematical reasoning. Furthermore, this knowledge might help the researchers and educators gain insight into metacognitive knowledge and students' mathematical reasoning.

2. Method

2.1. Participants

In total, 122 students of grade eight (51 males and 61 females) in four heterogeneous classrooms selected from Junior High School in Indonesia. The schools used a competency-based curriculum (Kemendikbud, 2016) that was part of the national curriculum for Junior High School in Indonesia.The classes were similar in student's mean age (M = 13.9; SD = 0.5), terms of size, and levels of mathematics achievement assessed from the prior to the beginning of the study. They had recently learned the geometry plane as the prior knowledge to teach solid geometry in this study.

2.2. Treatment

In this study, the paricipants studied mathematics under one of two conditions: collaborative learning embedded within metacognitive strategies (COLAB+META) and collaborative learning with no metacognitive strategies (COLAB). The two conditions were edentically in terms of: the mathematical tasks that must be done by the students, the material matter to be learned that was geometry, the lesson structure, and the number of hours' mathematics was taught (6 meeting). Under both conditions, each period included three parts: a) apperception activities guided by the teacher (15 minutes); (b) practicing activities discussion of learning subjects in small heterogeneus groups (55 minutes); and also (c) teacher's review of the main ideas of the lesson with the whole class (10 minutes).

In all four classes, the participants were exposed to the same tasks that were the reasoning mathematics tasks. None of the participants was introduced to the task prior from beginning of the study or during the strategies. Besides, learning of all four classes using small heterogeneous groups was composed of five participants: two low achievers, two middle achiever, and one high achiever. The participants were assigned to work in groups by the teacher according to their pre-test task. The differences of both treatments were only with regard to the metacognitive strategies.

COLAB+META condition used IMPROVE [21] to practice the participants to activate the metacognitive strategies in small groups. They were taught to formulate and answer four parts of self-questions: *comprehension, connection, strategic,* and *reflection*. The *comprehension questions* were designed to support the participants in reflect on the problem before solving it on the worksheet. The *comprehension questions*, each participant had to read, to undrestand, and to describe the problem. The *connections questions* were designed to support the participants to focus on differences and similarities between previous and new knowledge. The *strategic questions* were designed to support the participats considering strategies which would be chosen by the participant to solve the problem. The last question was *reflection questions* which were designed to support the participants to the

reflection of the processes and the solution. The participants used metacognitive strategies during the discussion process in small group guided by worksheet. When all participants agreed with the solution, they wrote it in their notebook.

When the participants followed COLAB did the learning with small heterogeneous groups, they did not use the self-questioning in their worksheet. Each group would read the worksheet aloud, tried to solve the problems and explained their reasoning. When they incorrect to solve the problem or they didn't agree with the solution, each group discussed the problem until a solution was achieved. When all group members agreed with the solution, every participant wrote it down in their notebook. When none of the group members knew how to solve the worksheet, they would ask to teacher.

2.3. Testing and Measurements

The present study was used two measurements, one for assessing the prior knowledge (pretest), and the other for the post-test. Each measurement was composed into three parts of the mathematical reasoning: (a) making conjecture (e.g., how long it takes to fulfill the volume of a beam-shaped bathtub whose size is known); (b) providing arguments (e.g., a beverage company wants to package its product in net 250ml, what can you say about the shape and surface area to minimize packaging cost); and (c) observing patterns (e.g., the following design is composed some cubes. The first cubes rib is *a*, the second ribs is *2a*, the thrid ribs is *4a*. If the surface area of the second cubes is known, find the surface area the fifth cubes). It should be noted that the questions of reasoning were never discussed during the classroom learning. For the sake of simplicity, all scores of measurements had been presented in terms of percent correct answers.

A 6-item at each pre-test and post-test was administered to all participants at the beginning of this study. Each part of mathematical reasoning represented 2-item. For each item, the students received a score for 0 (incorrect and no response) to 5 (full correct response). The interrater reliability of the scoring was checked by the mathematics teacher who was not part of this study. The reliability result of Kunder Richardson was $\alpha = 0.86$.

3. Result and Discussion

Four classes were formed under one of two condition (COLAB or COLAB+META) during teach of plane geometry. All students were then tested individually in three parts of mathematical reasoning. *Making conjecture*, *providing argments*, and observing *patterns* answers were scored separetely. Mean score and standard deviations on the test measuring are reported in Table 1.

Table 1. Mean score and standard deviations on the test measuring by prior knowledge and treatment.

Condition	Test Measuring	COLAB		COLAB+META	
		high-achiever N = 29	low-achiever N = 31	high-achiever N = 32	low-achiever N = 30
Pretest	M	23.97	20.87	23.48	20.93
	SD	11.52	12.43	10.32	12.45
Posttest	making conjecture				
	M	8.22	6.15	9.57	7.19
	SD	1.94	1.64	1.94	1.51
	providing argments				
	M	7.05	5.89	9.14	7.51
	SD	2.19	1.78	1.97	1.68
	observing patterns				
	M	7.76	5.78	9.44	7.88
	SD	1.83	2.24	2.12	1.96
	total				
	M	23.73	18.96	28.15	21.88
	SD	8.98	8.63	7.87	8.73

A two-way ANOVA was performed on pretest. One factor was the treatment (COLAB+META; COLAB) and the other prior knowledge. According to Tabel 1 of the pretest, there was no significant differences between the two treatments on prior knowledge (F(1,118) = 2,15, $p > 0.05$; M = 22.42 and M = 22.21; SD = 18.47 and SD = 18.58, for the COLAB and COLAB+META groups respectively). These results showed that the initial conditions of both groups for prior knowledge are the same.

According to Tabel 1 of the post-test, a two-way (treatment × prior-knowledge) Manova was conducted as the part of the mathematical reasoning. There was significant effect for the treatment F(3,116) = 4.83, $p < .05$, and for prior knowledge F(3,116) = 4.02, $p < .05$. However, there was no significant effect of the interaction between the treatment and prior knowledge F(3,116) = 2.31, $p > 0.05$. Results indicates that there is a differences in effectiveness between learning with COLAB and COLAB+META. As can be seen from Tabel 1, the interaction was due to a large difference mean score between COLAB and COLAB+META under the making conjecture (3.2); providing arguments (3.8); and observing patterns (4.96).

Higher and lower achievers got the benefits from the metacognitive strategies as the part of mathematical reasoning. It was analized by Effect-Size that were calculated according to Cohen's formula as the differences between the mean scores of the experimental (COLAB+META) and the control (COLAB) condition, then devided by the standard deviation of the control condition. *making conjecture* score was found for higher achieving students (0.69) and for lower achieving students (0.63). *Providing argments* score was found for higher achieving students (0.95) and for lower achieving students (0.91). *Observing patterns* score was found for higher achieving students (0.91) and for lower achieving students (0.93). This finding indicated that both higher and lower achievers got the benefit from the metacognitive strategies.

It could be seen that the using of metacognitive strategy in the experiment class was more superior that that in control class which did not use metacognitive strategy. In finishing the problem of mathematical reasoning, the students needed to analyse the relevant information of the problem, so they could arrange the strategy to solve the problem [22]. The metacognitive strategy seemed effectively to guide the students to identify correctly the relevant information of the problem. It caused by the using of self-questioning in the metacognitive strategy. Through the self-questioning, the students were asked to understand the problem by collecting the information then they thought the strategy to solve it. It could help them to solve the problem since by solving problem of mathematics reasoning the student's must can build the model of the problem given based on the whole information of the problem through reasoning skills [23]. The reasoning indicators, for example making conjecture and providing argument, would be helped in self-questioning where the students more expressed their ideas toward the gotten information during the process of solving problem.

Theoretically, giving a treatment toward two classes was as same as formed the groups and could happen the exchange of information and knowledge. Nevertheless, there were more ideas that were gotten during teaching and learning process in the experiment class caused by the self-questioning technique that was used in metacognitive strategy. This technique could guide the students to solve the problem by using correct strategy so that the discussion process could happen better than in control class which only focused on the solving problem. Besides, the students who were being active in the experiment class could increase the exchange of information and knowledge and more ideas that could be gotten so that the discussion process worked well than in the control class. The active discussion could increase achievement of the students' learning outcome of mathematics [24], and it automatically developed their students' reasoning [25].

In the point of fact in Tabel 1, mean in low-achiever in the COLAB+META condition performed significantly better than high-achiever in the COLAB for *providing arguments* and *observing patterns*. It indicated that the metacognitive strategies were intensive to lead lower achievers to make conclusions and arguments based on all of the information given in the computations and on the task they solved. There was at least the explanation that the students who learned by metacognitive strategies (IMPROVE) frequently used more logical-formal arguments [19]. Therefore, there was a

reason while studying the unit, the students that used COLAB+META not only had to explain their strategies to their peers and the reasons for using those strategies but also had to compare, contrast, and analyze the meaning of the problems and the solutions. The finding regarding the effects of metacognitive strategies on students' achievement extended previous findings ([26]–[28]). Furthermore, in this study adds to the already of evidence that lower achievers who learned to activate metacognitive processes were better able to solve mathematical reasoning than students who were not exposed to metacognitive strategies. Therefore, there was no superior effect that was found in collaborative learning of mathematics and no significant negative effects were found either. On the performance of *making conjecture* measures, the mean of COLAB in high-achiever tended to be higher than the mean of COLAB+META in low-achiever.

These results indicated that the students using metacognitive strategies treatment were better for them than without using metacognitive. In the class given treatment using *comprehension question* (e.g., "What is the problem all about?") which guided the students to search for all relevant information as well as distinguished between the irrelevant and relevant information. Furthermore, there was also the *connection questions* (e.g., "What are the differences between the problem at hand and the problems you have solved in the past?") that directed the students to pay attention to all of the information and the structures of the assigned tasks. In solving problem of mathematical reasoning, the students must be able to construct a model of the given problem on the basis of entire information given in the problem text [23]. Another self-question, *strategic questions,* had to describe the answers from *what* (e.g., "what strategy can be used for solve the problem?"), *why* (e.g., "why is this strategy most appropriate for solving problem?"), *how* (e.g., "how the last conclusions of the solution the problem?"), and asked the students to answer *why* questions during the solution processess of solving problems to help them elaborate and retain the infromation. As students explained and justified their thinking challenge to their peers and teacher, they also clarified their own thinking and became the owners of 'knowing' [29]. The last of the self-questions was *reflection questions,* which were designed to support the student to reflect on the processes and the solutions (e.g., "Does the strategy make sense?") in which the reflection questions could guide them to reflect on their own and their peers' solutions, so they must think firstly before solving the problem [30]. Besides, the reflection questions could guide the students to remind and strengthen their understanding. Having the strong understanding especially could make the students easily to solve the problem and also gave the influence for their learning outcome [31].

Therefore, there is reason to suppose that being trained use the self-questions making students easier in solving mathematical reasoning problems. This reason showed in Figure 1 that showed the mean score of pre-test where the result of the COLAB+META students (M = 25.02; SD = 8.30) was higher than that of COLAB students (M = 22.20; SD = 8.81).

	Prior-knowledge	Posttest
■ COLAB	22,42	22,2
■ COLAB+META	21,35	25,02

■ COLAB ■ COLAB+META
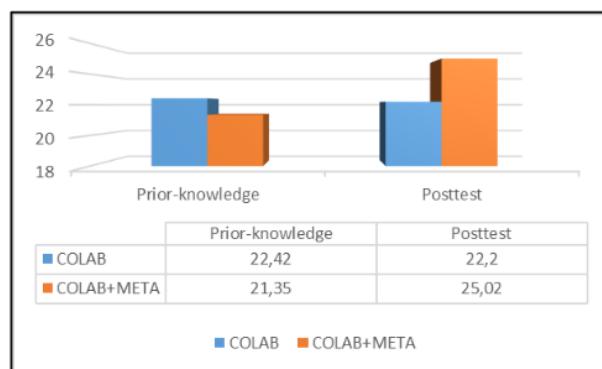
Figure 1. Overall mean score on the test measuring by prior-knowledge and treatment.

This finding made the hypothesis that metacognitive strategies embedded in collaborative mathematics classrooms that could be benefical for mathematical reasoning was confirmed. This study confirmed that the students who were exposed to metacognitive strategies attained a higher level of mathematical reasoning, and they were better able to explain their mathematical ideas in writing than before getting treatment. Nevertheless, we did not reject the collaborative learning as a learning process in learning mathematics or any other domain. As reported research [32] has demonstrated that collaborative learning gave more space and opportunities for students to discuss, to solve problems, to create solutions, to provide and to challenge ideas to their peers and teacher to support in learning mathematics.

The metacognitive effect had been shown to be very rebust over 42 years of research. Not just the reasoning that worked, some studies had found that metacognitive support on: threading comprehension [33]; critical thinking [34]; mathematical problem solving [35]; students' performance on context-based tasks [36]. In the present study, similar to previous studies, the students opportunity-to-learn with metacognitive were shown to be important in learning outcomes. Consistent with the success given in learning outcomes, we recomended the use of metacognitive in mathematics strategiess.

4. Conclusions and Future Work
Our findings indicated that integrating the metacognitive strategies could be a springboard to enhance the mathematical reasoning tasks. This paper had theoretical and practical implications. Theoretically, this study pointed toward the essential components of metacognitive and their effects on cognitive processes in learning mathematics. Practically, this study pointed toward the positive effects of integrating metacognitive strategies in enhancing mathematical reasoning ability.

In sum, this paper gave insight into metacognitive in the discplines of mathematics. Furthermore, this study could also be used as the material consideration in planning learning model or strategy that aimed to optimize and improve the students' ability in mathematics reasoning. These findings also called for the educators and researcher to train metacognitive strategies on different subject. The extent to which metacognitive strategies used in the present study were approriate for different subject that was not known yet nowadays and might be investigated in future research.

5. References
[1]   NCTM 2000 *Principles and standards for school mathematics* (Reston: NCTM)
[2]   Herbert S, Vale C, Bragg L A, Loong E and Widjaja W 2015 *Int. J. Educ. Res* **74** 26–37
[3]   Thompson P W, Hatfield N J, Yoon H, Joshua S and Byerley C 2017 *J. Math. Behav* **48** 95-100
[4]   Zaman A, Jumani N B, Ali A and Hussain M A 2011 *Procedia - Soc. Behav. Sci* **12** 588-594
[5]   Fujita T and Jones K 2014 *Int. J. Educ. Res* **64** 81-91
[6]   Moore K and Moore K C 2014 *J. Res. Math. Educ* **45** 102–138
[7]   Adu-Gyamfi K and Bossé M J 2014 *Int. J. Sci. Math. Educ* **12** 167–192
[8]   Ellis A B, Ozgur Z, Kulow T, Dogan M F and Amidon J 2016 *Math. Think. Learn* **18** 151–181
[9]   Lobato J and Siebert D 2002 *J. Math. Behav* **21** 87–116
[10]   Dogbey J 2016 *Int. J. Sci. Math. Educ* **14** 1175–1196
[11]   Bragg L A, Herbert S, Loong E Y K, Vale C and Widjaja W 2016 *Math. Educ. Res. J* **28**
[12]   Kramarski B and Zoldan S 2008 *J. Educ. Res* **102** 137–151
[13]   Hartman H J 2001 Developing Students' Metacognitive Knowledge and Skills *Metacognition in Learning and Instruction: Theory, Research and Practice* ed H J Hartman (Dordrecht: Springer Netherlands) pp 33–68
[14]   Strijbos J W and Wichmann A 2018 *Eur. J. Psychol. Educ* **33** 1–9
[15]   Prast E J, Van de Weijer-Bergsma E, Kroesbergen E H and Van Luit J E H 2018 *Learn. Instr* **54** 22–34
[16]   Dubé A K and McEwen R N 2015 *Learn. Instr* **40** 89–98
[17]   Kramarski B 2004 *Learn. Instr* **14** 593–619

[18]   Montague M 2008 *Learn. Disabil. Q* **31** 7–44
[19]   Mevarech Z R and Kramarski B 1997 *Am. Educ. Res. J* **2** 365–395
[20]   Kramarski B 2008 *Metacognition Learn* **3** 83–99
[21]   Mevarech Z R and Kramarski B 1997 *Am. Educ. Res. J* **34** 365–394
[22]   Kuo E, Hull M M, Gupta A and Elby A 2013 *Sci. Educ* **97** 32–57
[23]   Jeannotte D and Kieran C 2017 *Educ. Stud. Math* **96**
[24]   Kirschner F, Paas F, Kirschner P A and Janssen J 2011 *Learn. Instr* **21** 587–599
[25]   Mueller M, Yankelewitz D and Maher C 2014 *Investig. Math. Learn* **7** 1–20
[26]   Kramarski B and Mevarech Z R 2003 *Am. Educ. Res. J* **40** 281–310
[27]   Pennequin V, Sorel O, Nanty I and Fontaine R 2010 *Think. Reason* **16** 198–220
[28]   Maqsud M 1998 *Educ. Res* **40** 237–243
[29]   Chin C, Brown D E and Bruce B C 2002   *Int. J. Sci. Educ* **24** 521–549
[30]   Gagnière L, Betrancourt M and Détienne F 2012 *Revue Europeene de Psychologie Appliquee* **62** 3–81
[31]   Ayazgök B and Yalçin N 2014 *Procedia - Soc. Behav. Sci* **141** 774–780
[32]   Zakaria E, Chin L and Daud Y 2010 *J. Soc. Sci* **6** 272–275
[33]   Jones T and Brown C 2011 *Int. J. Instr* **4** 5–22
[34]   Ku K Y L and Ho L T 2010 *Metacognition Learn* **5** 251–267
[35]   Desoete A, Roeyers H and De Clercq A 2003 *J. Educ. Psychol* **95** 188–200
[36]   Wijaya A, Doorman M and Sciences B 2014 *Proc. Int. Cont. on Research, Implementation, and Education of Mathematics and Science (Yogyakarta)* pp 1–7

Enhancing an Ability Mathematical Reasoning through Metacognitive Strategies

12% SIMILARITY INDEX

2% INTERNET SOURCES

9% PUBLICATIONS

6% STUDENT PAPERS

1	Kramarski, B.. "Making sense of graphs: does metacognitive instruction make a difference on students' mathematical conceptions and alternative conceptions?", Learning and Instruction, 200412 Publication	5%
2	B. Kramarski. "How can self-regulated learning be supported in mathematical E-learning environments?", Journal of Computer Assisted Learning, 2/2006 Publication	1%
3	Bracha Kramarski, Adiva Liberman. "Chapter 3 Using electronic mail communication and metacognitive instruction to improve mathematical problem solving", Springer Science and Business Media LLC, 2003 Publication	1%
4	Submitted to Australian Catholic University Student Paper	1%

| 5 | tasued.edu.ng
Internet Source | 1% |

| 6 | Submitted to Nottingham Trent University
Student Paper | 1% |

| 7 | Submitted to Universitas Pendidikan Indonesia
Student Paper | <1% |

| 8 | link.springer.com
Internet Source | <1% |

| 9 | Submitted to Program Pascasarjana Universitas Negeri Yogyakarta
Student Paper | <1% |

| 10 | Submitted to University of New England
Student Paper | <1% |

| 11 | Submitted to North West University
Student Paper | <1% |

| 12 | Submitted to Western Governors University
Student Paper | <1% |

| 13 | Submitted to CSU, Hayward
Student Paper | <1% |

| 14 | issuu.com
Internet Source | <1% |

| 15 | Zemira R. Mevarech, Shirlei Terkieltaub, Tova Vinberger, Vered Nevet. "The effects of meta-cognitive instruction on third and sixth graders | <1% |

solving word problems", ZDM, 2010
Publication

16 Chaya Hirsch. "Effects of Computer Algebra System (CAS) with Metacognitive Training on Mathematical Reasoning", Educational Media International, 9/1/2003
Publication   <1 %

17 Bracha Kramarski, Nava Mizrachi. "Online Discussion and Self-Regulated Learning: Effects of Instructional Methods on Mathematical Literacy", The Journal of Educational Research, 2006
Publication   <1 %

| Exclude quotes | Off | Exclude matches | Off |
| Exclude bibliography | On | | |

Enhancing an Ability Mathematical Reasoning through Metacognitive Strategies

FINAL GRADE

/100

GENERAL COMMENTS

Instructor